

[MAe] Exam Summary 2023-04-04

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March 31th to April 1st 2023

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1 Learning Objectives

Definitions / Deductions:

- You are able to give and explain the definition of a power with exponent zero and the definition of powers with rational and real exponents.
- You know (actively) the power rules (with the restrictions).
- You know the properties of power functions: symmetries, domain and range, special point, kind of graph.

With all the solved exercises

2 General Definition

Let n be a natural number, then $\underbrace{a * a * \dots * a}_n = a^n$. a^n is the n^{th} power of a . a is called the base and n is called the exponent.

2.1 Special Cases

$$0^n = 0 (n \neq 0)$$

$$0^0 \text{ undefined}$$

$$1^n = 1$$

$$n^0 = 1 (n \neq 0)$$

$$(-1)^n = 1 \text{ if } n \text{ is even or } -1 \text{ if } n \text{ is odd.}$$

3 Power Rules

$$a^m * a^n = a^{m+n}$$

$$a^m : a^n = \begin{cases} a^{m-n}, & \text{if } m > n \\ 1, & \text{if } m = n \\ \frac{1}{a^{n-m}}, & \text{if } m < n \end{cases}$$

$$a^n * b^n = (a * b)^n$$

$$a^n : b^n = \left(\frac{a}{b}\right)^n$$

$$a^{-n} = \frac{1}{a^n} \quad (n \text{ a natural number})$$

$$\frac{1}{a^{-n}} = a^n \quad (n \text{ a natural number})$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

4 Powers with Real Exponents

A power with an irrational exponent can be approximated arbitrarily precisely. Then the rules above can be applied.

5 Glossary

5.1 Vertex of a Parabola

This is either the lowest or the highest point on the graph.

5.2 Asymptote

An asymptote is a line that a graph approaches.

5.3 Inflection Point

The point where the curvature changes its sign (i.e. direction).

5.4 Saddle Point

A saddle point is an inflection point where the slope of the curve is zero (the sign of the slope of the curve is preserved.)

5.5 Even Function

A function f that satisfies $f(-x) = f(x)$ for all values of x in its domain is called an even function. The graph of an even function is symmetric about the y-axis.

5.6 Odd Function

A function f that satisfies $f(-x) = -f(x)$ for all values of x in its domain is called an odd function. The graph of an odd function is symmetric about the origin.

6 Power Functions

6.1 Definition

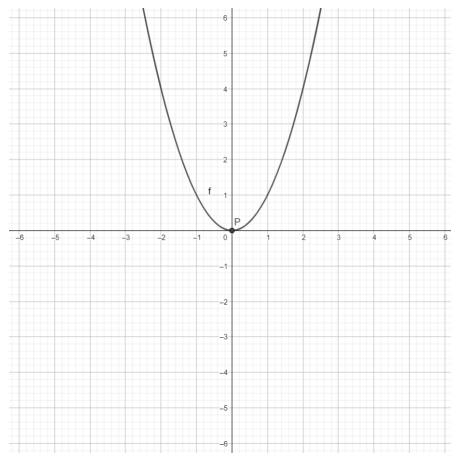
Functions that are of the kind: $f(x) = x^k, k \in \mathbb{Z}$ are called power functions.

6.2 General Function Equation

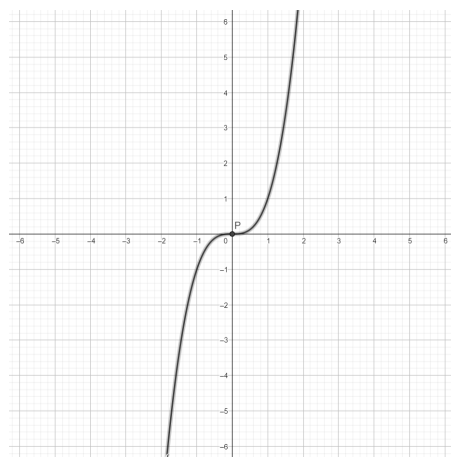
$$y = f(x) = a * (x - u)^k + v$$

6.3 Examples

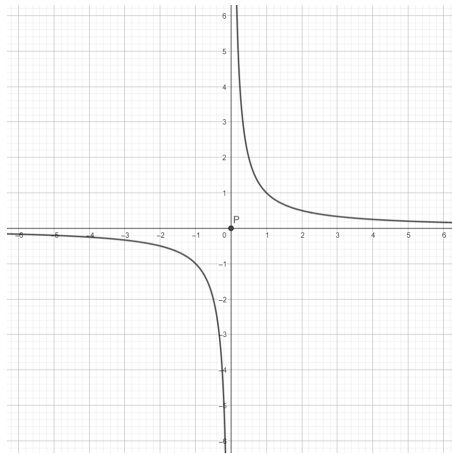
6.3.1 x^2



6.3.2 x^3



6.3.3 x^{-1}



6.3.4 x^{-2}

